Edgar E. Escultura and the inequality of 1 and 0.999... (rough translation)

One who has concerned himself with the 0.999... issue now for years already is Edgar E. Escultura. He regards 0.999... as being clearly smaller than 1, and accordingly believes the usual kind of arithmetic is incorrect. Therefore he advocates a system of his own where the difference of 1 and 0.999... is given by the *dark number* d*. Brightness does not appear to be Escultura's strongest point, but his stance leads us to some interesting reflections.

Commotion

On May 5, 2005 *The Manila Times* published a piece about a supposedly revolutionary discovery by the Philippine Professor Edgar E. Escultura. It concerned the so-called Last Theorem of Fermat. This theorem (actually a *conjecture* because Fermat did not supply us with his proof) has only recently been proved by the British mathematician Andrew Wiles. Professor Escultura now claimed the proof of Wiles to be incorrect, and Fermat's Theorem to be false. Further, Escultura claimed to have counterexamples *proving* Fermat Last Theorem incorrect [7]. A rather remarkable claim, because the proof of Wiles has been admired worldwide. Escultura's claim indeed seemed highly improbable, but not absolutely impossible. The original proof of Wiles (of 1993) did in fact contain an error, which he only managed to fix the following year (1994).

Fermat's Last Theorem

Fermat's Last Theorem looks simple enough: the following equation (1), according to this theorem, has no solutions for natural numbers n greater than 2 and positive natural numbers a, b and c.

(1) $a^n + b^n = c^n$.

Intuitionism and constructivism

Escultura cites no specific points at which Wiles would have made an error. Instead, he mentions long known points of criticism, already brought in against the regular practice of mathematics by proponents of the so-called constructivist and intuitionist philosophical viewpoints within the foundations of mathematics. These movements accept only concepts and proofs of which we can form a clear image. Mathematics is seen as an intellectual activity of individuals, and, in principle, only those results are accepted that can be concretely verified. In particular, working with infinite sets (when these are considered as completed and given) and the use of indirect forms of proof (where the existence of mathematical objects is shown without one being able to give an example), are criticized. The intuitionists and constructivists have, however, thus far been unable to give any hard evidence (in the form of logical contradictions) of the supposed defects of standard mathematics. Still, something may be be said on behalf of the criticisms given. Indeed, we often have the most confidence in those concepts and proofs of which the scope and meaning are absolutely clear. As we move further and further into unchartered territory, and go on arguing about things of which we are hardly able or just cannot form our selfes a clear image, our confidence in the results found declines. Even when the logic of our proofs seems impeccably... Escultura's criticisms sound quite similar, but add nothing to it. They moreover make a sloppy and scratchy impression, and thus do the constructivist and intuitionist standpoint no good. Escultura also thinks to have solved about all of the major problems of modern physics and mathematics. See for example the work of Escultura himself [6], [7], [8] and [9], and his polemics with Larry Freeman [10] and [11], and with the people of the Wikipedia [20]. Finally, Alecks P. Pabico in his piece *Anatomy of a hoax* [16] paints a not too friendly portrait of Escultura as a public figure. Nevertheless, a constructivist or intuitionist mathematics is indeed possible, although certain sections of mathematics as we know it have to go. In this area lots of work has already been done. A clear exposition of the intuitionist and constructivist position is given by Mame Maloney of the University of Chicago in the price-winning 'student paper' *Constructivism: A Realistic Approach to Math?* [14].

Escultura and Wiles

Escultura essentially claims that mathematics as commonly known - and in particular arithmetic - is wrong. Now the very advanced proof of Wiles nevertheless operates within the framework of mainstream mathematics. Therefore, according to Escultura, Wiles proof is doomed. As an alternative Escultura then presents his own number system: the new real numbers. Applied to these new numbers Fermat's Last Theorem appears to be false. And it is here that we see what is wrong with Escultura's claim. Fermat never meant his theorem to apply to Escultura's new numbers, but to the usual positive natural numbers: 1, 2, 3, 4, 5, ... Further, intuitionist and constructivist criticism does not and need not convince a mainstream mathematician. From the viewpoint of mainstream mathematics there is nothing wrong with mathematics as commonly understood. So we see, that hardly anything of Escultura's revolutionary discoveries remains.

New real numbers

Escultura sees his *new real numbers* as being essentially sequences of digits. Furthermore, he accepts as basic only those decimal numbers whose digits can in principle be calculated. To these he adds two additional objects, i.e. d* (*dark number*, the difference between 1 and 0.999...) and u* (*unbounded number* for infinite numbers). He believes to have achieved a major scientific breakthrough with this new real numbers. (Sometimes, he also allows new real numbers to have *arbitrary* digits. This is contrary to his constructivist position, but these arbitrary new real numbers kind of resemble the free choice sequences of the Dutch intuitionist mathematician L.E.J. Brouwer (1881-1966).)

Constructions

The study of computable numbers and computable functions, etc. is a recognized part of mathematics for years already. There are several closely related mathematical disciplines that occupy themselves with this kind of study. We have recursion theory, theoretical computer science and the theories of computable and constructive analysis. Computability can be defined in different ways, and many results have already been proven in this field. Furthermore, constructive analysis has shown - in particular through the work of the American mathematician Errett Albert Bishop (1928-1983) - that a large part of the usual form of analysis (integral and differential calculus) can be rebuild in a constructivist way (see [2]). But what I have read of Escultura does not help these branches of mathematics any further.

Decimal numbers

The idea of introducing the real numbers by means of their decimal representation is also not new. Already in the book *Vorlesungen über allgemeine Arithmetik* (from 1885) [19], dl. I, pp. 97-124) of the Austrian mathematician Otto Stolz (1842-1905) the real numbers are introduced in a similar way. He considers infinite sequences of rational numbers:

$\varphi_0 , \varphi_1 , \varphi_2 , \varphi_3 , \cdots , \varphi_n , \cdots$

With:

$\varphi_0 = c_0$
$\varphi_1 = c_0 + c_1 . 1/e$
$\varphi_2 = c_0 + c_1 . 1/e + c_2 . 1/e^2$
$\varphi_3 = c_0 + c_1 . 1/e + c_2 . 1/e^2 + c_3 . 1/e^3$
...
$\varphi_n = c_0 + c_1 . 1/e + c_2 . 1/e^2 + c_3 . 1/e^3 + ... + c_n . 1/e^n$
...

Where c_0 is an integer, where e is the so-called *base* or *radix* (a natural number greater than or equal to 2), and where $c_1 , c_2 , c_3 , ... , c_n , ...$ are the 'digits' (to be chosen from 0, 1, 2, ... , e-1).

When working in the decimal system we find for example for the famous number π the infinite sequence of rational numbers:

$\varphi_0 = 3$
$\varphi_1 = 3 + 1 .1/10$
$\varphi_2 = 3 + 1 . 1/10 + 4 .1/100$
$\varphi_3 = 3 + 1 . 1/10 + 4 .1/100 + 1 . 1/1000$
...

Alternatively written:

$\varphi_0 = 3$
$\varphi_1 = 3.1$
$\varphi_2 = 3.14$
$\varphi_3 = 3.141$
...

Stolz noted that such sequences satisfy the following property (E):

For any positive rational number ε (no matter how small) there is a natural number N such that:

$| \varphi_{n+r} - \varphi_n | < \varepsilon$ for all natural numbers n > N and all positive natural numbers r.

The addition and multiplication of these infinite sequences of rational numbers are defined termwise. The sum $\varphi + \psi$ and the product $\varphi.\psi$ of the following sequences:

$\varphi = \varphi_0 , \varphi_1 , \varphi_2 , \varphi_3 , \dots , \varphi_n , \dots$
$\psi = \psi_0 , \psi_1 , \psi_2 , \psi_3 , \dots , \psi_n , \dots$

thus are:

$\varphi + \psi = \varphi_0 + \psi_0 , \varphi_1 + \psi_1 , \varphi_2 + \psi_2 , \varphi_3 + \psi_3 , \dots , \varphi_n + \psi_n , \dots$
$\varphi . \psi = \varphi_0 \cdot \psi_0 , \varphi_1 \cdot \psi_1 , \varphi_2 \cdot \psi_2 , \varphi_3 \cdot \psi_3 , \dots , \varphi_n \cdot \psi_n , \dots$

Two such infinite sequences of rational numbers φ and ψ are considered equal precisely then when for any positive rational number ε (no matter how small) there is a natural number N such that:

$$|\varphi_n - \psi_n| < \varepsilon \quad \text{for } n > N .$$

As a consequence of this convention it holds also in the system of Stolz that 0.999... = 1. He then proves that also the sums and products of such sequences of rational numbers can be written as decimal numbers. By following this line of reasoning, it is eventually shown that the sequences of rational numbers with the property (E) identify the real numbers. Periodic decimal numbers correspond to the rational numbers and non-periodic to the irrationale numbers.

There are also more *modern* constructions of the real numbers based on their decimal (or binary) representation (see [12], [13] and [15]). Again - nothing new under the sun.

1 ≠ 0.999...

Remains Escultura's belief that 1 is unequal to 0.999... . These two numbers are equal within the usual system of real numbers, and this equality can indeed be proven within this system. However, the internet is full of discussions about whether 1 and 0.999... are *really* equal. This cannot easily be dismissed as a discussion of layman and pseudomathematicians. The American mathematician, computer scientist and writer Rudy Rucker reported in his popular science book *Infinity and the Mind* ([17] p. 79) that he regards the assumption of an infinitely small (infinitesimal) difference between 1 and 0.999... as not all that crazy. Unfortunately he does not further develop the idea. For further reading please see the Wikipedia. It has a very extensive and worth reading lemma on the 0.999... question, with many references [1].

[...]

Conclusion

Other than he himself thinks, Escultura's claims and theories add nothing substantial to what others have already done. This need not be a problem. We cannot all be at the top. More serious, however, are Escultura's carelessness and lack of logic. Mathematical and physical theories by themselves are often already complicated enough. If the reader on top of that has to puzzle what the writer means by a certain sentence, or what (apparently misprinted) symbols had to be printed here and there, the story soon enough becomes incomprehensible. Most readers then quit. Maybe a few will think themselves too stupid. But many would rather prefer to push Escultura's work aside as manifest nonsense. So Escultura will have to formulate his views and theories in a more comprehensible way. That is - if he indeed wants to keep an audience.

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[The original article can be found on http://wetenschap.infonu.nl/wiskunde/32165-edgar-e-escultura-en-de-ongelijkheid-van-1-en-0999.html .]

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